Interval Kalman Filtering Based Navigation for an Uninhabited Surface Vehicle

Amit Motwani
Supervisory Team:
Dr Sanjay Sharma, Prof Robert Sutton, Dr Phil Culverhouse

School of Marine Science and Engineering
Plymouth University
www.springer-usv.com
Outline

• Background: A USV called Springer
• Navigation, Guidance and Control, and the role of the Kalman Filter
• The Kalman Filter
• Simulation of Kalman Filter estimation of compass heading
• Interval Kalman Filtering
• Future directions
Background: A USV called Springer

USV  uninhabited surface vessel \{ remotely controlled autonomous surface vessel (ASV) \}

- Built at Plymouth University in 2004
- 4m long weighing approx 600kg
- max speed ~ 4m/s
- Steering: differential thrust, trolling motors
- A cost effective & environmentally friendly USV for pollutant tracking

ASV requires

Navigation, Guidance and Control system

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Navigation, Guidance and Control System, and the role of Kalman Filtering

- **Navigation:** the art of knowing where you are, how fast you are moving and in which direction (Stovall).

**Autonomous Navigation:** relies on **sensors** and some algorithm to process data.

- **Sensors:**
  - Accuracy
  - Noise
  - Faulty?

\[ \text{KF} \]  \quad \text{Optimal Estimate}

\[ \text{KF} \]  \quad \text{Data fusion}

\[ \text{KF} \]  \quad \text{Sensor Redundancy}

- **Navigational Suite:**
  - GPS
  - Compass
  - Gyro
  - Visual SLAM
  - Kalman filter
The continuation *Springer* project

Project consists of integrating the following:

- **Feature matching** algorithm based on **visual SLAM**
- Novel Navigation system based on fuzzy techniques and **interval Kalman filtering**
- Adaptive autopilot based on **closed loop identification and MPC**
The continuation *Springer* project: why?

- GPS signals are relatively weak
- Signals can be easily jammed
- Signal loss may occur in confined areas
- Need for robust marine NGC systems
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The Kalman Filter

- **KF** is a **state estimator** of a dynamic system.
- Obtains state estimate from **a priori model statistics** and **noisy measurements**.
  - **Optimal (MMSE)**
  - **Recursive** formulation

![Diagram of Compass Model (prior statistics)](image1)

Electronic Compass

**noisy reading**

Kalman Filter

### Optimal Estimate $\hat{\chi}$
The Kalman Filter

- KF is a **state estimator** of a dynamic system.
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  - Optimal (MMSE)
  - **Recursive** formulation
Simulation: Compass Heading

TCM2 magnetic compass:

\[ x_{k+1} = \begin{bmatrix} 0.2796 & 0.6971 \\ 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0.4364 & 0 \end{bmatrix}^T u_k + \omega_k \]

\[ z_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + v_k \]

\[ \omega_k \sim N(0, Q_k) \]

\[ v_k \sim N(0, R_k) \]
Simulation: Compass Heading

KF assumptions:
- Linear, known model
  \[ x_{k+1} = A \, x_k + B \, u_k + \omega_k \]
  \[ z_k = H \, x_k + \nu_k \]
- White, Gaussian noise models
  \[ \omega_k \sim N(0, Q) \quad \nu_k \sim N(0, R) \]

Effect of erroneous model: three simulations showing the effect of increasing modelling error
Interval Kalman Filtering

- KF: lump all uncertainty as a probabilistic disturbance
- IKF: include bounded uncertainty in model parameters:

Interval System

\[
\begin{align*}
    x_{k+1}^I &= A^I x_k^I + B^I u_k + \omega_k \\
    z_k &= H_k^I x_k + \nu_k
\end{align*}
\]

some parameters are not known exactly but known to lie within a certain interval

\[
M_k^I = [M_k - \Delta M_k, M_k + \Delta M_k], \quad M \in \{A, B, H\}
\]

components made up of intervals rather than point values

Nominal System

\[
x_{k+1} = \begin{bmatrix} 0.2796 & 0.6971 \\ 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0.4364 \\ 0 \end{bmatrix} u_k + \omega_k
\]

Interval System

\[
x_{k+1} = \begin{bmatrix} [0.2796-\delta, 0.2796+\delta] \\ 1 \end{bmatrix} x_k + \begin{bmatrix} 0.6971 \\ 0 \end{bmatrix} u_k + \begin{bmatrix} 0.4364 \\ 0 \end{bmatrix} u_k + \omega_k
\]
Interval Kalman Filtering

\[
\begin{align*}
x_{k+1} &= A_k x_k + B_k u_k + \omega_k \\
z_k &= H_k x_k + v_k
\end{align*}
\]

IKF is derived on the same principles as KF, retaining statistical optimality and recursive structure.
Interval Kalman Filtering

The IKF gives upper and lower boundaries of the optimal estimates of every point-valued system included in the interval system.

$x$ (state)

$k$ (time step)

upper boundary

lower boundary

real state
Measurement
KF Estimate
IKF Estimate
Interval Kalman Filtering: Interval Arithmetic

- Interval computation must be carried out with care, e.g. non-distributivity

\[ x(y + z) \neq xy + xz \]

and also singularity problem for dividing by interval containing zero.

- Implemented on computers using external rounding: each Real number is represented by a small interval, by rounding down and rounding up.

- Tools used: INTLAB (for MATLAB), available since 1998, which uses BLAS routines for interval operations.
Interval Kalman Filtering

\[
x_{k+1} = \begin{bmatrix} 0.2796 & 0.6971 \\ 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0.4364 \\ 0 \end{bmatrix} u_k + \omega_k \quad \text{nominal system - KF}
\]

\[
x_{k+1} = \begin{bmatrix} 0.2796 + \delta & 0.6971 \\ 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0.4364 \\ 0 \end{bmatrix} u_k + \omega_k \quad \text{real system - real state}
\]

\[
x_{k+1} = \begin{bmatrix} [0.2796 - \delta, 0.2796 + \delta] & 0.6971 \\ 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0.4364 \\ 0 \end{bmatrix} u_k + \omega_k \quad \text{interval system - IKF}
\]

\[\delta = 2\% \text{ of nominal value}\]

KF Divergence due to imprecise modelling
Interval Kalman Filtering

\[
\begin{align*}
    x_{k+1} &= \begin{bmatrix} 0.2796 & 0.6971 \\ 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0.4364 \\ 0 \end{bmatrix} u_k + \omega_k0.5
    \text{nominal system - KF} \\
    x_{k+1} &= \begin{bmatrix} 0.2796 + \delta & 0.6971 \\ 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 0.4364 \\ 0 \end{bmatrix} u_k + \omega_k \\
    \text{real system - real state}
\end{align*}
\]

IKF Expanding boundaries, due to conservative nature of interval arithmetic.
Interval Kalman Filtering: Limiting the Boundaries

Resetting the intervals (state estimate and error covariance) to their mid-points when the state estimate interval exceeds a threshold T.

IKF estimate does not improve KF estimate.
Interval Kalman Filtering: Limiting the Boundaries

Reset in this case consists of limiting intervals to a certain percentage of their original widths (ξ).

IKF estimate centred around true value.
Interval Kalman Filtering: Limiting the Boundaries

- $T = 60$  
  $\xi = 50\%$

- $T = 55$  
  $\xi = 50\%$

- $T = 500$  
  $\xi = 0\%$

- $T = 5000$  
  $\xi = 0\%$
Conclusions and Future Work

- IKF can overcome divergence of KF due to modelling discrepancies.

- Raw implementation of IKF is impracticable due to rapidly increasing intervals.

- Limiting mechanisms therefore become necessary. These mechanisms typically interfere with IKF’s robust nature.

- Mechanisms implemented here are extremely sensitive to choice of adequate parameters. Therefore methods for optimal autonomous tuning of these parameters is desirable.

- Other limiting mechanisms that do not inhibit IKF robustness are sought, possibly using AI-based strategies.